P.E

Universo $= \Omega = \{\ldots\}$

acontecimento certo $\{\ldots\} = \Omega$

" simples $\{\omega\}$

" impossível $\{\emptyset\}$

10.1)

1.1) $^{25}A_3 = \frac{25!}{22!} = 25\times24\times23 = 13800$

1.2) $^{25}C_3 = \frac{25!}{22!\,3!} = \frac{25\times24\times23}{6!} = 2300$

1.3) $^{4}A_4 = \frac{4!}{0!} = 4! = 24$

1.4) 4 M
6 F
2 Q

a) $^{4}A_4\,^{6}A_6\,^{2}A_2\,^{3}A_3 = 4!\,6!\,2!\,3! = 207360$

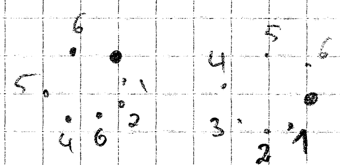
b) $^{9}A_9\,^{4}A_4 = 9!\,4! = 8709120$

pensar em 9 livros, onde um deles é o conjunto dos livros de matemática logo depois multiplica-se pelas suas combinações

1.5) $^{12}A_7 = \frac{12!}{5!} = 12\times11\times10\times9\times8\times7\times6 = 3991680$

1.6) $^{10}A_4 = \frac{10!}{6!} = 10\times9\times8\times7 = 5040$

1.7) $11! = 39916800$



=> em sequência

1.8) $3\times2 = 6$

1.9) a) $^{8}C_6 = \frac{8!}{2!\,6!} = 28$

b) $^{8}C_5 \cdot {}^{2}C_1 = \frac{8!}{3!\,5!} \cdot 2! = 112$

1.10) a) $^{12}C_2 = \frac{12!}{10!\,2!} = \frac{12\times11}{2} = 66$

b) $^{11}C_1 = \frac{11!}{10!\,1!} = 11$

c) $^{12}C_3 = \frac{12!}{9!\,3!} = \frac{12\times11\times10}{6} = 220$

d) $^{11}C_2 = \frac{11!}{9!\,2!}\quad \frac{11\times10}{2} = 55$

e) ${}^{10}C_1 = 10$

1.11) a) $\binom{4}{1}\binom{48}{9}$

b) $\binom{52}{10} - \binom{48}{10}$

Ao total de possibilidades tiro todas onde não saiem nenhum ás, então ficamos com as que sairem pelo menos 1 ás

c) $\binom{4}{2}\binom{48}{8}$

d) $\binom{52}{10} - \binom{48}{10} - \binom{4}{1}\binom{48}{9}$

e) $\binom{16}{1}\binom{36}{9}$

1.12) a) $\frac{(n-2)!}{(n-2-2)!\,2!} = 6 \Leftrightarrow (n-2)! = 12(n-4)! \Leftrightarrow (n-2)(n-3) = 12$

$\Leftrightarrow 12 = n^2 - 5n + 6 \Leftrightarrow n^2 - 5n - 6 = 0 \Leftrightarrow \frac{5 \pm \sqrt{25 - 4 \cdot 1 \cdot -6}}{2}$

$\Leftrightarrow \frac{5 \pm \sqrt{49}}{2} = \frac{5 \pm 7}{2} = \cancel{-1} \vee 6$

b) $\frac{(n+1)!}{(n-1)!\,2!} - \frac{(n-1)!}{(n-3)!\,2!} - \frac{n!}{(n-2)!\,2!} = -1$

$\Leftrightarrow \frac{(n+1)n\cancel{(n-1)!}}{\cancel{(n-1)!}\,2!} - \frac{(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}\,2!} - \frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}\,2!} = -1$

$\Leftrightarrow \frac{(n+1)n - (n-1)(n-2) - n(n-1)}{2} = -1$

$\Leftrightarrow \cancel{n^2} + n - n^2 + 2n + n - \cancel{2} - \cancel{n^2} + n = -\cancel{2}$

$\Leftrightarrow -n^2 + 5n = 0$

c) $5 \cdot \frac{n!}{(n-3)!\,3!} = \frac{(n+2)!}{(n-2)!\,4!}$

$\Leftrightarrow 5 \cdot \frac{\cancel{n(n-1)}(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}\,3!} = \frac{(n+2)(n+1)\cancel{n(n-1)}\cancel{(n-2)!}}{\cancel{(n-2)!}\,4!}$

$\Leftrightarrow 5(n-2)\,24 = (n+2)(n+1)\,6$

$\Leftrightarrow 120n - 240 = (n^2 + n + 2n + 2)6$

$\Leftrightarrow 120n - 240 = 6n^2 + 6n + 12n + 12$

$\Leftrightarrow -6n^2 + 102n - 252 = 0 \Rightarrow n = 3$

Definiçăo Axiomatica de Probabilidade

A probabilidade é uma funçăo, que a cada acontecimento A faz correspon-der um valor real $P(A)$, e que verifica os seguintes axiomas:

1. $P(A) \geq 0$, qualquer que seja o acontecimento A
2. $P(\Omega) = 1$
3. Se $A \cap B = \emptyset$ entăo $P(A \cup B) = P(A) + P(B)$

A em 1º lugar   A em 2º lugar

1.13) a) $\frac{{}^2A_2 + {}^1A_1}{{}^3A_3} = \frac{2! + 1!}{3!} = \frac{3}{6} = \frac{1}{2}$

b) $\frac{{}^2A_2}{{}^3A_3} = \frac{2!}{3!} = \frac{2}{6} = \frac{1}{3}$

1.14) a) $\frac{4}{7} \times \frac{3}{6} = \frac{2}{7}$

b) $\frac{3}{7} \cdot \frac{2}{6} = \frac{6}{42} = \frac{1}{7}$

b) nenhuma ser nova

c) $1 - \frac{1}{7} = \frac{7-1}{7} = \frac{6}{7}$

1.15) $\frac{\overset{I\,I}{8 \cdot 7} + \overset{T\,I}{7 \cdot 8} + \overset{F\,F}{5 \cdot 4} + \overset{F\,T}{5\,7} + \overset{T\,F}{7 \cdot 5} + \overset{T\,T}{7 \cdot 6} + \overset{I\,T}{8 \cdot 7}}{20 \cdot 19} = \frac{15}{19}$

1.16) a) $\frac{4}{17} \cdot \frac{5}{16} \cdot \frac{3}{15} \cdot \frac{5}{14} = \frac{30\cancel{0}}{5712\cancel{0}} \approx 0{,}005$

b) $0{,}005 \times 4! = 0{,}126$

~~1.17~~

② $A \subseteq B \Rightarrow P(A) \le P(B)$

$B = A \cup (B-A) \qquad A \cap (B-A) = \emptyset$

$P(B) \overset{3}{=} P(A) + \underbrace{P(B-A)}_{\substack{\geq 0 \\ 1}} \Rightarrow P(B) \geq P(A)$

⑤ $P(A-B) = P(A\cap \bar{B}) = P(A) - P(A\cap B)$

$A = (A\cap B) \cup (A\cap \bar{B})$

$(A\cap B) \cap (A\cap \bar{B}) = \emptyset$

$P(A) \overset{3}{=} P(A\cap B) + P(A\cap \bar{B}) \Leftrightarrow P(A) - P(A\cap \bar{B}) = P(A\cap B)$

$P(A\cap B) = P(A|B)\,P(B)$

$P(A\cap B) = P(B|A)\,P(A)$

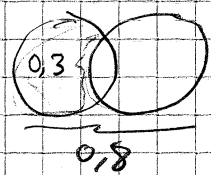
se A e B independentes

$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{P(A)\cancel{P(B)}}{\cancel{P(B)}} = P(A) \quad , P(B) > 0$

1.25) $P(A\cap \bar{B}) = P(A) - P(A\cap \bar{B}) = P(A) - P(A)P(B)$

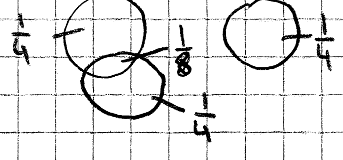
$= P(A)(1-P(B)) = P(A)P(\bar{B})$

1.17) $P(A\cup B) = 0{,}8$
$P(A-B) = 0{,}3$



$P(B) = 0{,}8 - 0{,}3 = 0{,}5$

1.18) $P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap C)$



$= \frac{3}{4} - \frac{1}{8} = \frac{6-1}{8} = \frac{5}{8}$

1.19) $P(A-B) = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$

$P(A\cup B) = \frac{2}{3} + \frac{1}{2} - \frac{1}{3} = \frac{1}{3} + \frac{1}{2} = \frac{5}{6}$

$$P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B}) = 1 - P(A \cap B) = 1 - \frac{1}{3} = \frac{2}{3}.$$

$$P(\bar{A} \cap B) = P(B - A) = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

$$P(A \cup \bar{B}) = P(A) + P(\bar{B}) - P(A \cap \bar{B}) = \cancel{\frac{2}{3}} + 1 - \frac{1}{2} - \cancel{\frac{2}{3}} + \frac{1}{3}$$

$$= 1 - \frac{1}{2} + \frac{1}{3} = \frac{6 - 3 + 2}{6} = \frac{5}{6}$$

1.20) $P(V) = \frac{1}{2}$

$P(M) = \frac{3}{10}$

$P(V \cap M) = \frac{1}{10}$

a) $P(V \cup M) = \frac{1}{2} + \frac{3}{10} - \frac{1}{10} = \frac{5+2}{10} = \frac{7}{10}$

b) $P(\bar{V} \cap \bar{M}) = P(\overline{V \cup M}) = 1 - \frac{7}{10} = \frac{3}{10}$

1.21) a) $P(H \cap M) \overset{\text{independentes}}{=} P(H) \cdot P(M) = \frac{2}{3} \times \frac{3}{5} = \frac{2}{5}$

b) $P(H \cap \bar{M}) = \frac{3}{5} \cdot \frac{1}{3} = \frac{1}{5}$

c) $P(\bar{H} \cap M) = \frac{2}{5} \cdot \frac{2}{3} = \frac{4}{15}$

d) $P(H \cap \bar{M}) + P(\bar{H} \cap M) = \frac{1}{5} + \frac{4}{15} = \frac{7}{15}$

1.22) a) $P(C) = 40\%$

$P(L) = 30\%$

$P(C \cap L) = 15\%$

$P(C|L) = \frac{P(C \cap L)}{P(L)} = \frac{15\%}{30\%} = \frac{1}{2}$

$P(L|C) = \frac{P(C \cap L)}{P(C)} = \frac{15\%}{40\%} = 0{,}375\%$

b) $P(C \cup L) = 40 + 30 - 15 = 55\%$

Exercício

Sendo B um acontecimento fixo de um espaço de resultados Ω, tal que $P(B) > 0$, mostre que $P(\cdot|B)$ é uma probabilidade.

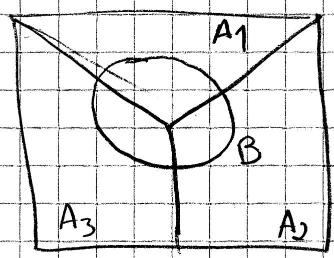
1. $P(A|B) = \frac{\overbrace{P(A \cap B)}^{\geq 0}}{\underbrace{P(B)}_{>0}} \geq 0 \quad \forall A$

2. $P(\Omega|B) = \frac{P(\Omega \cap B)}{P(B)} = 1$

3. Se $A_1 \cap A_2 = \emptyset \Rightarrow P(A_1 \cup A_2 | B) = P(A_1|B) + P(A_2|B)$

$$P(A_1 \cup A_2 | B) = \frac{P((A_1 \cup A_2) \cap B)}{P(B)} = \frac{P((A_1 \cap B) \cup (A_2 \cap B))}{P(B)}$$

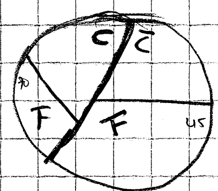
$$= \frac{P(A_1 \cap B) + P(A_2 \cap B)}{P(B)} = P(A_1|B) + P(A_2|B)$$



$$P(B) = P(B \cap \Omega)$$
$$= P(B \cap (\bigcup_{i=1}^{3} A_i))$$
$$= P((B \cap A_1) \cup (B \cap A_2) \cup (B \cap A_3))$$
$$= P(B \cap A_1) + P(B \cap A_2) + P(B \cap A_3)$$
$$= P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + P(B|A_3)P(A_3)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

1.23) $P(C) = 10\%$
$P(F|C) = 70\%$
$P(F|\bar{C}) = 45\%$



a) $P(F) = P(F|C)P(C) + P(F|\bar{C})P(\bar{C})$

$$= \frac{7}{10} \times \frac{1}{10} + \frac{4{,}5}{10} \cdot \frac{9}{10} = 0{,}475\%$$

b) $P(C|F) = \frac{P(F \cap C)}{P(F)} = \frac{P(F|C)P(C)}{P(F)} = \frac{\frac{7}{10} \cdot \frac{1}{10}}{0{,}475} = 0{,}147\%$

1.24) $P(A) = \frac{4}{15}$; $P(M) = \frac{2}{5}$; $P(D) = \frac{1}{3}$

$P(G|A) = \frac{3}{4}$; $P(G|M) = \frac{1}{2}$; $P(G|D) = \frac{1}{5}$

a) $P(G) = P(G|A)P(A) + P(G|M)P(M) + P(G|D)P(D)$

$$= \frac{4}{15} \cdot \frac{3}{4} + \frac{2}{5} \times \frac{1}{2} + \frac{1}{3} \cdot \frac{1}{5} = \frac{7}{15}$$

b) $P(M|G) = \frac{P(M \cap G)}{P(G)} = \frac{P(G|M) \cdot P(M)}{P(G)} = \frac{\frac{1}{5}}{\frac{7}{15}} = \frac{3}{7}$

1.29) $P(A) = \frac{2}{5}$; $P(B) = \frac{1}{10}$; $P(O) = \frac{9}{20}$; $P(AB) = \frac{1}{20}$

$P(E|A) = \frac{1}{50}$; $P(E|B) = \frac{3}{100}$; $P(E|O) = \frac{1}{20}$; $P(E|AB) = \frac{1}{10}$

a) $P(E) = \frac{2}{5} \cdot \frac{1}{50} + \frac{1}{10} \cdot \frac{3}{100} + \frac{9}{20} \cdot \frac{1}{20} + \frac{1}{20} \cdot \frac{1}{10} = 0{,}0385\%$

b) $P(AB|E) = \frac{P(AB \cap E)}{P(E)} = \frac{\frac{1}{20} \cdot \frac{1}{10}}{0{,}0385} = 0{,}1299\ \%$

1.26) $\binom{5}{2} \cdot 0{,}6^2 \cdot 0{,}4^3 = 0{,}2304$

1.27) $P(A) = 0{,}05$
$P(B) = 0{,}01$

a) $P(A \cap B) \overset{\text{independentes}}{=} P(A)P(B) = 0{,}05 \times 0{,}01 = 0{,}0005\ \%$

b) $P(A \cap \bar{B}) = P(A)P(\bar{B}) = 0{,}05 \times 0{,}99 = 0{,}0495\ \%$

c) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0{,}05 + 0{,}01 - 0{,}0005 = 0{,}0595\%$

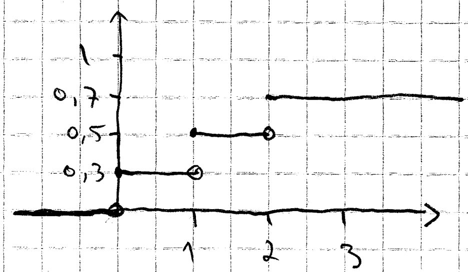
1.28) a) $\frac{2}{8} + \frac{1}{7} = \frac{1}{28}$ $\quad P(C|V) = 1 \quad P(V) = \frac{1}{4}$

b) $P(C) = \frac{3}{4} \cdot \frac{1}{2^3} + \frac{1}{4} = \frac{11}{32}$ $\quad P(C|\bar{V}) = \frac{1}{2} \quad P(\bar{V}) = \frac{3}{4}$

c) $P(V|C) = \frac{P(V \cap C)}{P(C)} = \frac{P(C|V)P(V)}{P(C)} = \frac{\frac{1}{4}}{\frac{11}{32}} = \frac{8}{11}$

2.1) a) $q + 0{,}3 = 0{,}5 \Leftrightarrow q = 0{,}2 \Rightarrow p = 0{,}3$

b) $F_X(x) = \begin{cases} 0 & , x < 0 \\ 0{,}3 & , 0 \leq x < 1 \\ 0{,}5 & , 1 \leq x < 2 \\ 0{,}7 & , 2 \leq x < 3 \\ 1 & , x \geq 3 \end{cases}$



c) $Y = \begin{cases} 0 & 40 & 80 & 120 \\ 0{,}3 & 0{,}2 & 0{,}2 & 0{,}3 \end{cases}$

$W = \begin{cases} 1 & 2 & 3 \\ 0{,}5 & 0{,}2 & 0{,}3 \end{cases}$ $\quad P(W=1) = P(X \leq 1)$

2.2) a) i) $P(X \leq 3) = 1/4$

ii) $P(1 < X \leq 2) = P(X \leq 2) - P(X \leq 1) = \frac{1}{4} - \frac{1}{6} = \frac{1}{12}$

iii) $P(2 \leq X < 6) = P(X < 6) - P(X < 2) = \frac{7}{12} - \frac{1}{6} = \frac{5}{12}$

iv) $P(|X-2| \leq 1) = P(1 \leq X \leq 3) = P(X \leq 3) - P(X < 1)$

$= \frac{1}{4} - 0 = \frac{1}{4}$

b) $X = \begin{cases} 1 & 2 & 4 & 5 & 6 \\ \frac{1}{6} & \frac{1}{12} & \frac{1}{4} & \frac{1}{12} & \frac{5}{12} \end{cases}$

c) Não. $P(X=1) \neq P(X=2)$

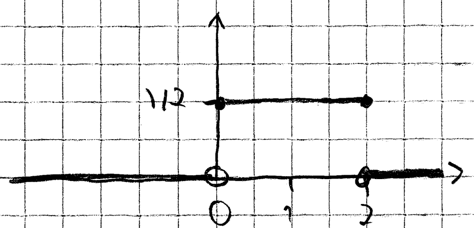
d) $P(X \geq 6 \mid X \geq 4) = \frac{P((X \geq 6) \cap (X \geq 4))}{P(X \geq 4)} = \frac{P(X \geq 6)}{P(X \geq 4)} = \frac{\frac{5}{\cancel{12}}}{\frac{9}{\cancel{12}}} = \frac{5}{9}$

2.3) a) $P(X > 100) = 1 - P(X \leq 100) = 1 - 0,6 = 0,4$

b) $P(X = 200 \mid X > 100) = \frac{P((X=200) \cap (X > 100))}{P(X > 100)} = \frac{0,1}{0,4} = \frac{1}{4}$

Seja X uma ~~var~~ v.a. contínua, com função densidade dada por

$f(x) = \begin{cases} 1/2 & , \text{se } x \in [0;2] \\ 0 & , \text{c.c} \end{cases}$



$P(0 \leq X \leq 1) = \int_0^1 f(x)\,dx = \frac{1}{2}$

$P(X \leq \frac{1}{2}) = \int_0^{1/2} f(x)\,dx = \frac{1}{4}$

densidade $\underset{()'}{\overset{\int}{\rightleftarrows}}$ distribuição

$P(X = a) = 0$

2.4) a)

1. $f(x) \geq 0, \forall_x \in \mathbb{R}$

para $x \in [-1; 0[$:

$k + x \geq 0 \Leftrightarrow k \geq -x \geq 0$

para $x \in [0; 1[$:

$k - x \geq 0 \Leftrightarrow k \geq x \geq 0$

2. $\int_{-\infty}^{+\infty} f(x)\,dx = 1$

$\int_{-\infty}^{+\infty} f(x)\,dx = \int_{-1}^{0} k+x\,dx + \int_{0}^{1} k-x\,dx = \left[x\left(k+\frac{x}{2}\right)\right]_{-1}^{0} + \left[x\left(k-\frac{x}{2}\right)\right]_{0}^{1}$

$= k - \frac{1}{2} + k - \frac{1}{2} = 2k - 1$

$2k - 1 = 1 \Rightarrow k = 1$

b) Função distribuição da v.a. X para $x < -1$, $F(x) = P(X \leq x) = 0$

para $-1 \leq x < 0$, $F(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)\,dt = \int_{-1}^{x} 1+t\,dt = \left[t + \frac{t^2}{2}\right]_{-1}^{x}$

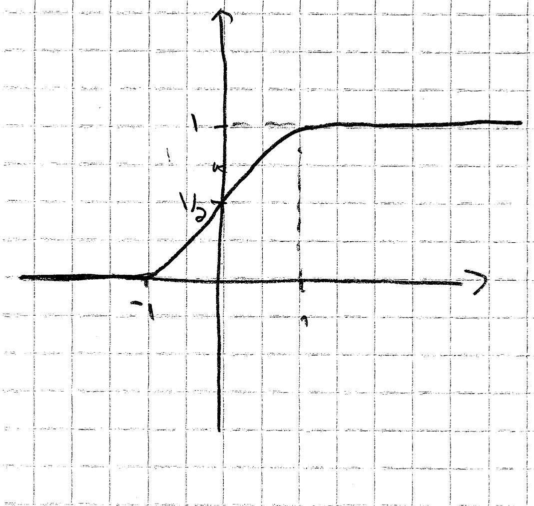
$= x + \frac{x^2}{2} + 1 - \frac{1}{2} = x + \frac{x^2}{2} + \frac{1}{2}$

para $0 \leq x < 1$, $F(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)\,dt = \int_{-1}^{0} (1+t)\,dt + \int_{0}^{x} (1-t)\,dt$

$= \left[t + \frac{t^2}{2}\right]_{-1}^{0} + \left[t - \frac{t^2}{2}\right]_{0}^{x} = +1 + \frac{1}{2} + x - \frac{x^2}{2} = x - \frac{x^2}{2} + \frac{1}{2}$

para $x \geq 1$, $F(x) = 1$

$$F(x) = \begin{cases} 0 & , x < -1 \\ \frac{(x+1)^2}{2} & , -1 \leq x < 0 \\ 1 - \frac{(x-1)^2}{2} & , 0 \leq x < 1 \\ 1 & , x \geq 1 \end{cases}$$



c) $P(X>0) = 1 - P(X \leq 0) = 1 - \left(1 - \frac{(0-1)^2}{2}\right) = \frac{1}{2}$

d) $P(X>0{,}5 \mid X>0) = \frac{P((X>0{,}5) \cap (X>0))}{P(X>0)} = \frac{P(X>0{,}5)}{P(X>0)}$

$= \frac{1 - P(X \leq 0{,}5)}{1 - P(X \leq 0)} = \frac{1 - 0{,}875}{1 - \frac{1}{2}} = \frac{1}{4}$

2.5) a) $f(x) \geq 0 \quad \forall x \in \mathbb{R}$

$\frac{a}{2} + \frac{a}{2} = 1 \Rightarrow a = 1$

$\frac{2b \times \frac{1}{2}}{2} + \frac{b}{2} = 1 \Rightarrow b = 1$

b) $\frac{b(b+a)}{2} = 1 \Rightarrow b(b+a) = 2$

2.6) a) 1. $f(x) \geq 0 \quad \forall x \in \mathbb{R}$

para $x \in \left]0 ; k\right[$:

$4x \geq 0 \Rightarrow x \geq 0$ entao $k \geq 0$

2. $\int_{-\infty}^{+\infty} f(x)\,dx = 1$

$\int_0^k 4x\,dx = \left[2x^2\right]_0^k = 2k^2 \qquad 2k^2 = 1 \Leftrightarrow k = \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2}$

b) para $x < 0$, $F(x) = P(X \leq x) = 0$

para $0 \leq x < \frac{\sqrt{2}}{2}$, $F(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)\,dt = \int_0^x 4t\,dt = \left[2t^2\right]_0^x = 2x^2$

para $x \geq \frac{\sqrt{2}}{2}$, $F(x) = 1$

$$F(x) = \begin{cases} 0, & x < 0 \\ 2x^2, & 0 \leq x < \frac{\sqrt{2}}{2} \\ 1, & x \geq \frac{\sqrt{2}}{2} \end{cases}$$

c) $P(1/4 \leq x \leq 1/3) = P(X \leq \frac{1}{3}) - P(X \leq \frac{1}{4}) = F(\frac{1}{3}) - F(\frac{1}{4})$

$2\left(\frac{1}{3}\right)^2 - 2\left(\frac{1}{4}\right)^2 = \frac{7}{72}$

$P(X \leq \chi_{p;0,5}) = p \Leftrightarrow F(\chi_{0,5}) = 0,5$

$\Leftrightarrow 2\chi_{0,5}^2 = 0,5 \Leftrightarrow \chi_{0,5} = \sqrt{\frac{0,5}{2}} \Leftrightarrow \chi_{0,5} = \frac{1}{2}$

$P(X \leq \chi_{0,95}) = 0,95 \Leftrightarrow 2\chi_{0,95}^2 = 0,95 \Leftrightarrow \chi_{0,95} = \sqrt{\frac{0,95}{2}} = 0,69$

d) $F_Y(y) = P(Y \leq y) = P(X-1 \leq y) = P(X \leq y+1) = F_X(y+1)$

$$F_Y(y) = \begin{cases} 0 & , y < -1 \\ 2(y-1)^2 & , -1 \leq y < \frac{\sqrt{2}}{2} - 1 \\ 1 & , y \geq \frac{\sqrt{2}}{2} - 1 \end{cases}$$

$F_T(t) = P(T \leq t) = P(X^3 \leq t) = P(X < \sqrt[3]{t}) = F_X(\sqrt[3]{t})$

$$F_T(t) = \begin{cases} 0 & , t < 0 \\ 2t^{2/3} & , 0 \leq t < \frac{2^{3/2}}{2} \\ 1 & , t \geq 2^{-3/2} \end{cases}$$

2.7) a) 1. $f(x) \geq 0 \ \forall x \in \mathbb{R}$

para $x \geq 0$

$Ke^{-\frac{x}{100}} \geq 0 \Rightarrow K \geq 0$

2. $\int_{-\infty}^{+\infty} f(x)dx = 1$

$\int_0^{+\infty} Ke^{-\frac{x}{100}} dx = 1 \Leftrightarrow \int_0^a Ke^{-\frac{x}{100}} dx = -100K\left[e^{-\frac{x}{100}}\right]_0^a = -100K\left(e^{-\frac{a}{100}} \xrightarrow{} 0 \ -1\right) = 100K$

$100K = 1 \Leftrightarrow K = \frac{1}{100}$

$P(50 \leq X \leq 150) = \int_{50}^{150} \frac{e^{-\frac{x}{100}}}{100} dx = -\left[e^{-\frac{x}{100}}\right]_{50}^{150} = -\left(e^{-\frac{150}{100}} - e^{-\frac{50}{100}}\right) = 0,3834$

b) $P(X<100) = \int_{0}^{100} \frac{1}{100}e^{-\frac{x}{100}}\,dx = -\left[e^{-\frac{x}{100}}\right]_0^{100} = -\left(e^{-1}-e^{0}\right)$

$= -\frac{1}{e}+1 = 0,6321$

c) $P(X>200 \mid X>100) = \dfrac{P\left((X>200)\cap(X>100)\right)}{P(X>100)}$ $\qquad \int_0^{200} f(x)\,dx = -\left(e^{-2}-1\right) =$

$= \dfrac{P(X>200)}{P(X>100)} = \dfrac{1-P(X\le 200)}{1-P(X\le 100)} = \dfrac{1-(-e^{-2}+1)}{1-(-e^{-1}+1)} = 0,3679$

2.8) a) 1. $f(x)\geq 0 \ \forall x\in\mathbb{R}$

para $\boxed{-1<x\le 0}$

$c(1+x)\geq 0 \Rightarrow c\geq 0$

para $\boxed{0<x<2}$

$c\left(1-\frac{x}{2}\right)\geq 0 \Rightarrow c\geq 0$

2. $\int_{-\infty}^{+\infty} f(x)\,dx = 1 \Leftrightarrow \int_{-1}^{0} c(1+x)\,dx + \int_{0}^{2} c\left(1-\frac{x}{2}\right)dx = 1$

$\Leftrightarrow c\left[x+\frac{x^2}{2}\right]_{-1}^{0} + c\left[x-\frac{x^2}{4}\right]_0^2 = 1 \Leftrightarrow c\left[-\left(-1+\frac{1}{2}\right)\right] + c\left[2-1\right] = 1$

$\Leftrightarrow c\left(1-\frac{1}{2}\right) + c = 1 \Leftrightarrow 2c-\frac{c}{2} = 1 \Leftrightarrow c = \frac{2}{3}$

para $x\le -1$, $F(x) = P(X\le x) = 0$

para $-1<x\le 0$, $F(x) = P(X\le x) = \int_{-\infty}^{x} f(t)\,dt = \int_{-1}^{x} \frac{2}{3}(1+t)\,dt$

$= \frac{2}{3}\left[t+\frac{t^2}{2}\right]_{-1}^{x} = \frac{2}{3}\left(x+\frac{x^2}{2}-\left(-1+\frac{1}{2}\right)\right) = \frac{2}{3}\left(x+\frac{x^2}{2}+\frac{1}{2}\right)$

para $0<x<2$, $F(x) = P(X\le x) = \int_{-\infty}^{x} f(t)\,dt = \int_{-1}^{0}\frac{2}{3}(1+t)\,dt + \int_{0}^{x}\frac{2}{3}\left(1-\frac{t}{2}\right)dt$

$= \frac{2}{3}\left[t+\frac{t^2}{2}\right]_{-1}^{0} + \frac{2}{3}\left[t-\frac{t^2}{4}\right]_0^x = \frac{2}{3}\left(1+\frac{1}{2}\right) + \frac{2}{3}\left(x-\frac{x^2}{4}\right) = 1+\frac{2}{3}\left(x-\frac{x^2}{4}\right)$

para $x\geq 2$ $F(x) = P(X\le x) = 1$

$$F(x) = \begin{cases} 0 & , x \leq -1 \\ \frac{2}{3}\left(x + \frac{x^2}{2} + \frac{1}{2}\right) & , -1 < x \leq 0 \\ 1 + \frac{2}{3}\left(x - \frac{x^2}{4}\right) & , 0 \leq x \leq 2 \\ 1 & , x > 2 \end{cases}$$

b) $P(X \leq 0 \mid X \leq 1) = \dfrac{P((X \leq 0) \cap (X \leq 1))}{P(X \leq 1)} = \dfrac{P(X \leq 0)}{P(X \leq 1)}$

$$= \frac{\frac{1}{3}}{1 + \frac{2}{3}\left(1 - \frac{1}{4}\right)} = \frac{2}{9}$$

X v.a discreta	Y v.a contínua
$X = \begin{cases} 1 & 2 & 3 & 4 \\ 0{,}2 & 0{,}3 & 0{,}3 & 0{,}2 \end{cases}$	$f_Y(x) = \begin{cases} \frac{1}{3} & , x \in [0,3] \\ 0 & , \text{c.c.} \end{cases}$
$E(X) = 1 \times 0{,}2 + 2 \times 0{,}3 + 3 \times 0{,}3 + 4 \times 0{,}2$	$E(Y) = \int_{-\infty}^{+\infty} x f_Y(x)\, dx$
$= \sum_{i=1}^{4} x_i \times P(X = x_i)$	$= \int_0^3 \frac{x}{3}\, dx = \frac{3}{2}$

$E(aX + b) = aE(X) + b$

Seja X ~~v.a~~ v.a. discreta

$$E(X) = \sum_{i=1}^{\infty} x_i P(X = x_i)$$

$$E(aX + b) = \sum_{i=1}^{\infty} (ax_i + b) P(X = x_i)$$

$$= \sum_{i=1}^{\infty} \left[ax_i P(X = x_i) + b P(X = x_i)\right]$$

$$= \sum_{i=1}^{\infty} a x_i P(X = x_i) + \sum_{i=1}^{\infty} b P(X = x_i)$$

$$= aE(X) + b$$

$V(x) = E(x^2) - E^2(x)$; $V(x) = E((x-\mu)^2)$

$A\sigma(x) = \sqrt{V(x)}$

$V(ax+b) = a^2 V(x)$

$$V(ax+b) = E\left[\left((ax+b) - (a\mu+b)\right)^2\right]$$
$$= E\left[(ax - a\mu)^2\right]$$
$$= E\left[a^2(x-\mu)^2\right]$$
$$= a^2 E\left[(x-\mu)^2\right]$$
$$= a^2 V(x)$$

$$CV(x) = \frac{\sigma(x)}{|E(x)|} \times 100$$

2.9) $x = \begin{cases} 0 & 1 & 2 & 3 \\ 1/8 & 3/8 & 3/8 & 1/8 \end{cases}$

$E(x) = \frac{3}{8} + 2\cdot\frac{3}{8} + 3\cdot\frac{1}{8} = 3/2$

$E(x^2) = \frac{3}{8} + 2^2\cdot\frac{3}{8} + 3^2\cdot\frac{1}{8} = 3$

$V(x) = 3 - (3/2)^2 = 3 - \frac{9}{4} = \frac{3}{4}$

$E(x^3) = \frac{3}{8} + 2^3\cdot\frac{3}{8} + 3^3\cdot\frac{1}{8} = 6{,}75$

$E\left(\frac{1}{1+x}\right) = 1\cdot\frac{1}{8} + \frac{1}{2}\cdot\frac{3}{8} + \frac{1}{\cancel{3}}\cdot\frac{\cancel{3}}{8} + \frac{1}{4}\cdot\frac{1}{8} = 15/32$

2.10) $x = \begin{cases} 0 & 1 & 2 & 3 \\ 1/4 & p/2 & 5/8 - p/2 & 1/8 \end{cases}$, $0 \le p \le \frac{1}{2}$

$E(x) = \frac{p}{2} + \frac{10}{8} - p + \frac{3}{8} = -\frac{p}{2} + \frac{13}{8}$

$E(x^2) = \frac{p}{2} + \frac{20}{8} - 2p + \frac{9}{8} = -\frac{3}{2}p + \frac{29}{8}$